A COMPARISON OF TWO PLASMA MODELS

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Abstract

The time dependent behavior of a plasma which surrounds a highly biased conducting sphere is considered. The plasma is treated as either a cold two component fluid or as a warm plasma whose time development can be found by solving the Vlasov equation. Both models demonstrate oscillatory behavior, but the electric fields predicted by the models are quantitively quite different in regions close to the surface of the sphere and very similar otherwise. A broadening of the electron distribution function with time is observed indicating local heating of the plasma near the surface of the sphere.

Introduction

A problem of particular interest at a time when the production of high powered space systems is imminent is the determination of how surfaces which are biased at a potential well above or below plasma ground interact with the ambient plasma. An example of such a problem is the analysis of the return current from a plasma after the surface potential of a structure in space has been violently changed by an arc, which is essentially a short circuit to plasma ground. In seeking to understand the dynamic behavior of space plasmas what is needed is a description of the average behavior that emerges when a sea of charged particles interacts electromagnetically with itself and with any object immersed in it. A mathematically succinct if numerically unsolvable set of equations exists which captures the dynamics of plasma interactions in regions of low density where two particle correlations may be safely ignored. These are the Vlasov-Maxwell equations. In the case presently under consideration in which no magnetic field exists these equations are:

$$\frac{\partial f_s(\vec{r},\vec{v},t)}{\partial t} = -\vec{v}\cdot\vec{\nabla} f_s(\vec{r},\vec{v},t) - \vec{a}_s\cdot\vec{\nabla}_v f_s(\vec{r},\vec{v},t) \tag{1}$$

$$\vec{a}_s = \frac{q_s}{m_s}\vec{E} \tag{2}$$

$$\frac{\partial \vec{E}(\vec{r},t)}{\partial t} = -\frac{\Sigma \vec{j}_s(\vec{r},t)}{\epsilon} \tag{3}$$

$$\vec{j}_s(\vec{r},t) = q_s\int \vec{v} f_s(\vec{r},\vec{v},t)\,d^3v \tag{4}$$

In the above s refers to any plasma species. (1) is the Vlasov equation and (3) is Ampere's law which gives the time dependence of the electric field in the absence of magnetic fields. Ampere's law is coupled to the Vlasov equation through (4) which gives the current carried by the plasma constituents. The distribution function f(r,v,t) gives the probability of finding a plasma particle in an infinitesimal volume of phase space. Knowledge of this function permits one to calculate any characteristic of the plasma. For instance, the integration of f(r,v,t) over velocity space gives the plasma density, and the integration of the product of f(r,v,t) and velocity over velocity space gives the currents in the plasma (ref. 1). The Vlasov equation is unfortunately a seven dimensional quantity which makes finding its solution in the general case numerically intractable, (however, see ref. 2).

A simplification occurs when it is recognized that only quantities averaged over velocity space are of interest. These averages are found by calculating the moments of the Vlasov equation. The first two moment equations are:

$$\frac{\partial\rho(\vec{r},t)}{\partial t} = -\vec{\nabla}\cdot(\rho\vec{v}) \tag{5}$$

$$\frac{\partial\vec{v}}{\partial t} = -\vec{v}\cdot\vec{\nabla}\vec{v} + \frac{q}{m}(\vec{E}) - \frac{\vec{\nabla}\cdot\boldsymbol{P}}{\rho}\frac{q}{m} \tag{6}$$

P is the pressure tensor and is the charge density. To complete the description a complete set of moment equations is neccessary to predict the time evolution of the pressure tensor and related quantities. If the set of moment equations can be truncated then a fluid description of plasma behavior is possible (ref. 1,3,4). Otherwise one must solve the more fundamental coupled Vlasov-Maxwell equations themselves. In this paper both of these approaches will be used to look at the relatively simple case of a highly biased conducting sphere immersed in a plasma with no magnetic fields present.

The Cold Plasma

In the case where the electrostatic field energy is much larger than the plasma thermal energy it is reasonable to treat the plasma as a cold plasma. In this case the pressure tensor vanishes and equations (5) and (6) form a self contained set of equations if the electric field is known. The electric field is found by solving (3) with:

$$\vec{j}_s = \rho_s\vec{v}_s \tag{7}$$

Equations (3),(5),(6), and (7) form the numerical basis of the cold plasma model. Unfortunately in their present differential form these equations are very difficult to use near charged surfaces. One plasma species or another will certainly be repelled from such a surface and there will be a region, however thin, where the density of this species will become vanishingly small. Equation (6) decouples the velocity from the density of each species. To solve this equation near a surface will require the assignment of an average velocity to a region of space in which there are no particles. The arbitrariness of this assignment will be propagated into regions where particles exist and the calculation will lose accuracy, perhaps catastrophically. To avoid this dilemma a control volume approach is used. The integrals:

$$4\pi\frac{\partial\int\rho(\vec{r},t)r^2dr}{\partial t} = -\int_{surface}\rho(\vec{r},t)\vec{v}\cdot d\vec{a} \tag{8}$$

$$4\pi\frac{\partial\int\rho\vec{v}r^2dr}{\partial t} = -\int_{surface}\rho\vec{v}\vec{v}\cdot d\vec{a}+4\pi\int\rho\vec{a}r^2dr \tag{9}$$

give the change in charge density due to charge flow into and out of a region, and the change in current density due to the current flow into and out of a region and due to accelerations which occur within that region. Equations (8) and (9) are evaluated within and on concentric spherical shells. This results in:

$$\frac{\Delta\rho r^2}{\Delta t} = \frac{\rho_i r_i^2 v_i - \rho_o r_o^2 v_o}{\Delta r} \tag{10}$$

$$\frac{\Delta\rho r^2 v}{\Delta t} = \frac{\rho_i r_i^2 v_i^2 - \rho_o r_o^2 v_o^2}{\Delta r} + \rho r^2 a \tag{11}$$

(i and o refer to the quantities along the inner and outer radii respectively, of a given shell). The current density found by solving equation (11) is divided by the charge density (eqn. 10) to give the average velocity.

First order upwind differencing is used to evaluate these difference equations with special attention paid to regions where velocity reversal occurs to ensure that the differencing scheme is always conservative (ref. 5,6). The calculations were done in Fortran on a Mac IICX with an Apple/Unix attachment.
One of the interesting features of the cold plasma model is displayed in figure 1. This graph shows the current collected on the surface of a 1 meter sphere which was initially biased at +140 volts with respect to a neutral hydrogen plasma. The surface charges by attracting electrons to itself. After awhile, the rate of charging begins to decrease as the accumulating electrons begin to repel those that follow behind. Soon the induced field is strong enough to repel all approaching electrons and the repelled electrons expand outward until the cloud of ions left behind begins to pull them back again. The swarm of re-attracted electrons strikes the surface, increasing its negative charge and turning back further incoming electrons. These repelled electrons stream out into space again and the process repeats itself at a rate determined by the inverse of the electron plasma frequency.

The Warm Plasma

There is no angular dependence in any quantity under consideration. This greatly reduces the complexity of the Vlasov equation and it becomes amenable to numerical solution. In spherical coordinates:

$$f(\vec{r},\vec{v},t) = f_\theta(v_\theta)f_\phi(v_\phi)f_r(r,v_r,t) \tag{12}$$

$$\frac{\partial f_\theta(v_\theta)}{\partial t} = \frac{\partial f_\phi(v_\phi)}{\partial t} = 0 \tag{13}$$

$$\frac{\partial f_r(r,v_r,t)}{\partial t} = -v_r \frac{\partial f_r(r,v_r,t)}{\partial r} - a_r \frac{\partial f_r(r,v_r,t)}{\partial v_r} \tag{14}$$

Equation (1) is now a one dimensional equation and the coupled set of equations (1), (2), (3), (4) may now be solved numerically using upwind differencing. The strategy is as follows:

The electric field at time step n is used to find the distribution function at time step n+1. This is used in equation (4) to find the current density at time step n+1. This current is then used in equation (3) to find the electric field at time step n+1, (backward differencing), and the process is repeated to project the solution from time step n+1 to time step n+2. The boundary values at the surface of the sphere and at the last grid point are calculated by linear extrapolation from the two neighboring grid points, except that at the surface of the sphere there are assumed to be no emitted or backscattered plasma particles and the distribution function vanishes for all positive values of radial velocity.

The results of this calculation can be seen in figures (2-5), and (6-9). Figures (2-5) indicate that the warm and cold plasma models agree in their predicted electric values only at points at least 2.5 meters from the boundary of the sphere. Within that distance the fields are out of phase and the warm plasma model fields have less amplitude and are more rounded. The time evolution of the electron probability distribution function at the tenth grid point is shown in figures (6-9). They reveal two interesting characteristics. The distribution function does not retain its Gaussian shape, becoming peaked alternately on its left and then its right hand side, and once again oscillations at about the electron plasma frequency occur.

Conclusions

Two conclusions are self evident. The warm and cold plasma models give dissimilar results near the surface of the sphere even when the initial electric field is very strong. Figure 10 indicates that some of the electrostatic field energy initially available may be converted into thermal energy as the electrons flow back and forth in the the field induced by their own motion. This possibility is not allowed in a cold plasma model. The oscillatory nature of the solutions should also be noted. With no damping mechanism in either model no steady state solution exists, and a longitudinal electric field which oscillates at the electron plasma frequency is set up. This is in agreement with the results predicted by a linearized cold fluid model.

The Vlasov equation is a tantilizing object. Its solution gives complete information about the plasma including its internal pressure gradients and currents. That solution, however, can only be found in a limited number of situations. But one such case occurs when a longitudinal electric field is established around a simple object, and understanding the interactions between such fields and the plasma is necessary for understanding spacecraft charging.

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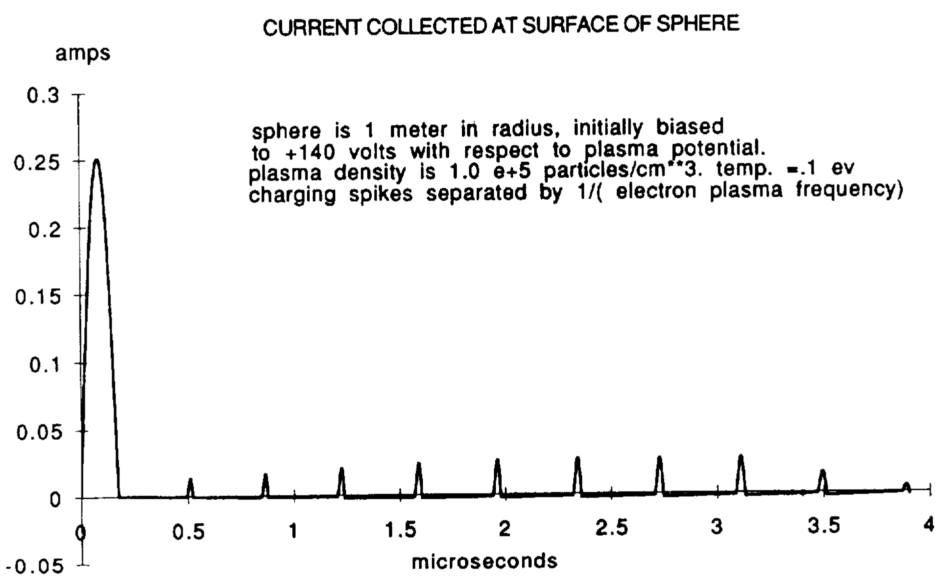


CURRENT COLLECTED AT SURFACE OF SPHERE
amps
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
sphere is 1 meter in radius, initially biased
to +140 volts with respect to plasma potential.
plasma density is 1.0 e+5 particles/cm**3. temp. =.1 ev
charging spikes separated by 1/(electron plasma frequency)
0
0.5
1
1.5
2
2.5
3
3.5
4
microseconds

fig. 1

ELECTRIC FIELD COMPARISONS

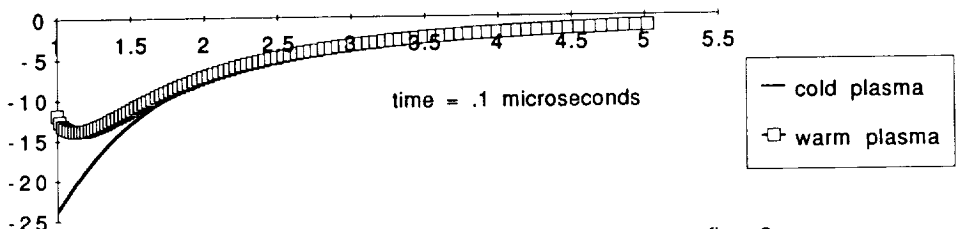


fig. 2

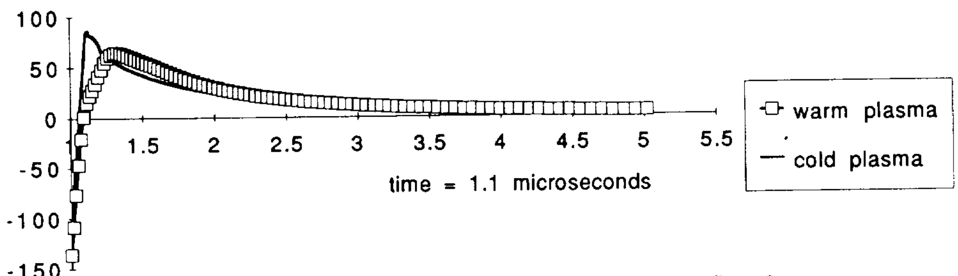


fig. 3

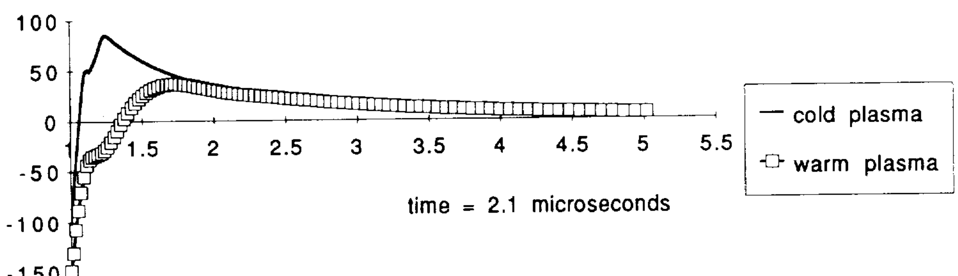


fig. 4

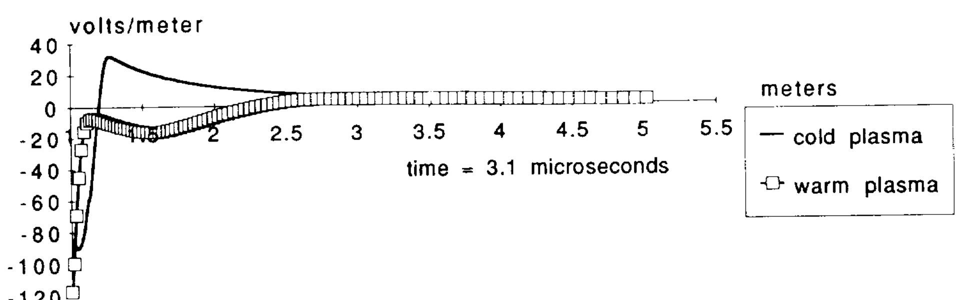


fig. 5

ELECTRON DISTRIBUTION FUNCTION AT 10th GRID POINT

fig. 6

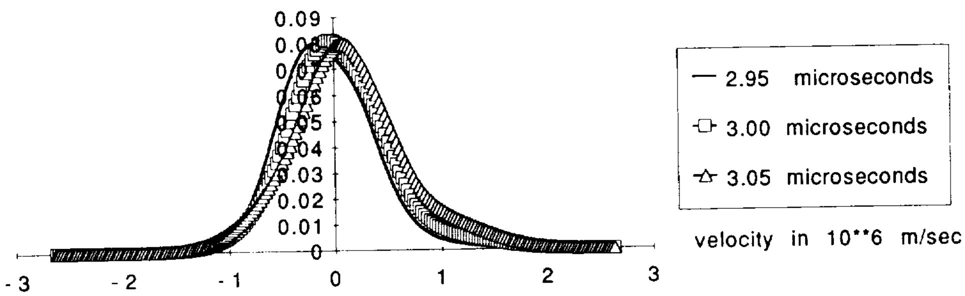


fig. 7

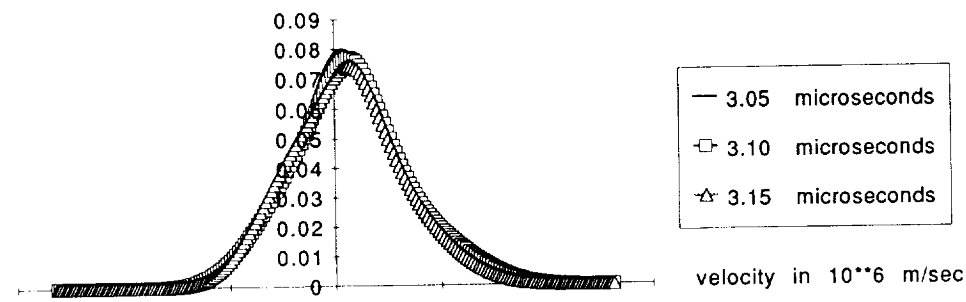


fig. 8

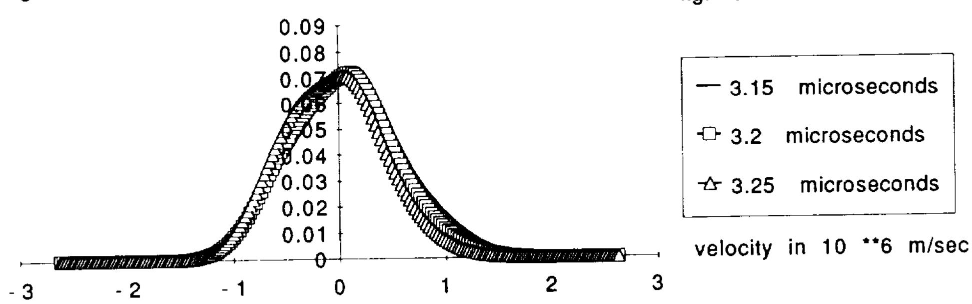
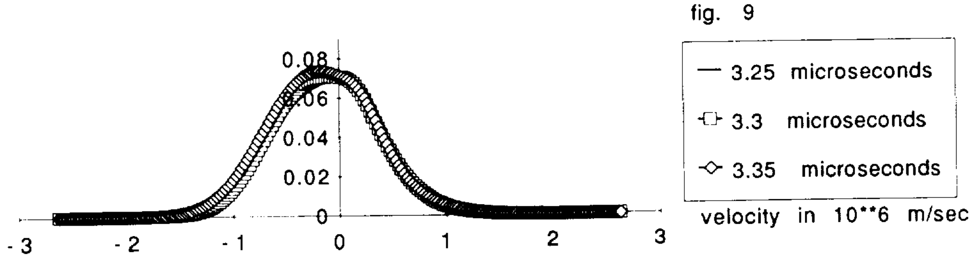


fig. 9



DISTRIBUTION FUNCTION BROADENING

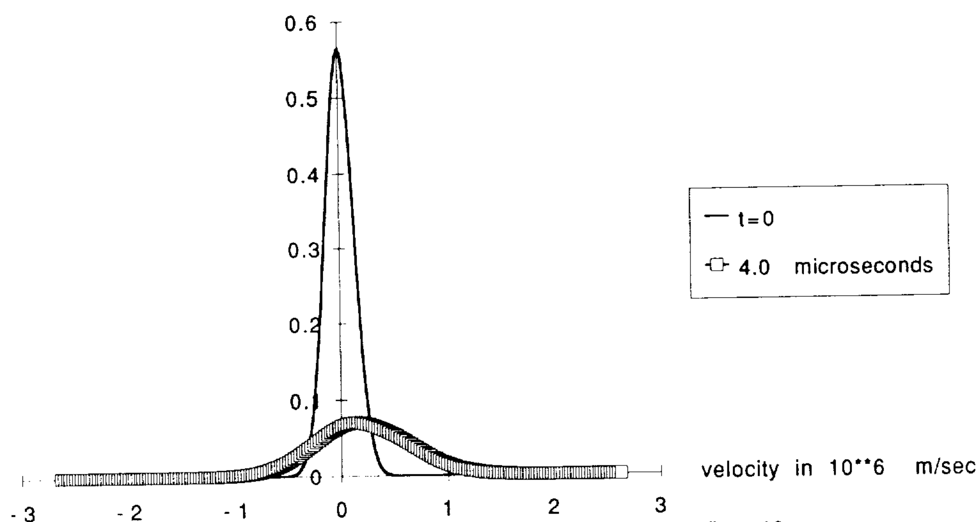


fig. 10

Erratum

The author has discovered that the warm plasma model discussed in this paper does not include angular momentum effects correctly. A corrected version has been constructed, and will be made available upon request.